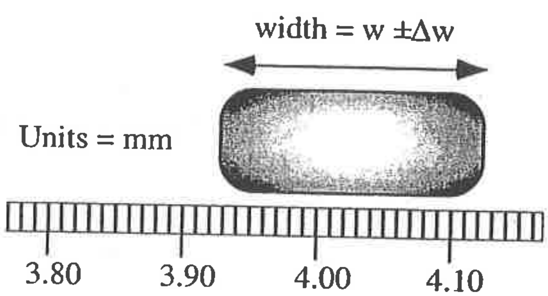
# UNCERTAINTY PROBLEMS

1. Four students measure the same length of string and their results are as follows:

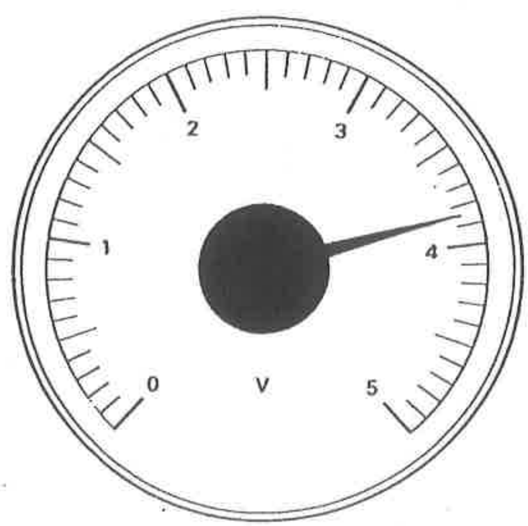
$$\ell_1 = 38.6\text{cm},\ \ell_2 = 38.7\text{cm},\ \ell_3 = 38.6\text{cm},\ \ell_4 = 38.5\text{cm}$$

Repeated measurement of the same quantity can improve the overall acceptable value of that measurement. What are the *mean* and its *absolute uncertainty* for the length of string?

2. What is the *width* and its *absolute uncertainty* of the object being measured in the sketch below?




3. The analogue voltmeter below measures a voltage on a scale of zero to 5 volts. What is the measured *voltage* and what is the *absolute uncertainty* shown here?



4. The digital stopwatch was started at a time $t_0 = 0$ and then was used to measure ten swings of a simple pendulum to a time of $t = 17.26\,\text{s}$.

| 1 | 7 | . | 2 | 6 |
|---|---|---|---|---|

If the time for ten swings of the pendulum is $17.26\,\text{s}$ what is the ***minimum absolute uncertainty*** in this measurement?

What is the *time* (period T) *of one complete pendulum swing* and its *absolute uncertainty*?

5. Add the following lengths: $12.2\text{cm} + 11.62\text{cm} + 0.891\text{cm}$

6. Add the following distances and express your result in units of centimeters (cm).

$$1.250 \times 10^{-3}\text{m} + 25.62\text{cm} + 426\text{mm}$$

7. Average speed is the ratio of distance to time: $v = \frac{s}{t}$.

What is the *average speed* if $s = 4.42\,\text{m}$ and $t = 3.085\text{s}$?

8. Given two masses, $m_1 = (100.0 \pm 0.4)\text{g}$ and $m_2 = (49.3 \pm 0.3)\text{g}$, what is their sum, $m_1 + m_2$, and what is their difference, $m_1 - m_2$, both expressed with uncertainties?

9. With a good stopwatch and some practice, you can measure times ranging from about a second up to many minutes with an uncertainty of 0.1 second or so. Suppose that we wish to find the period $\tau$ of a pendulum with $\tau \approx 0.5\,\text{s}$. If we time one oscillation, we will have an uncertainty of about 20%, but by timing several successive oscillations, we can do much better.

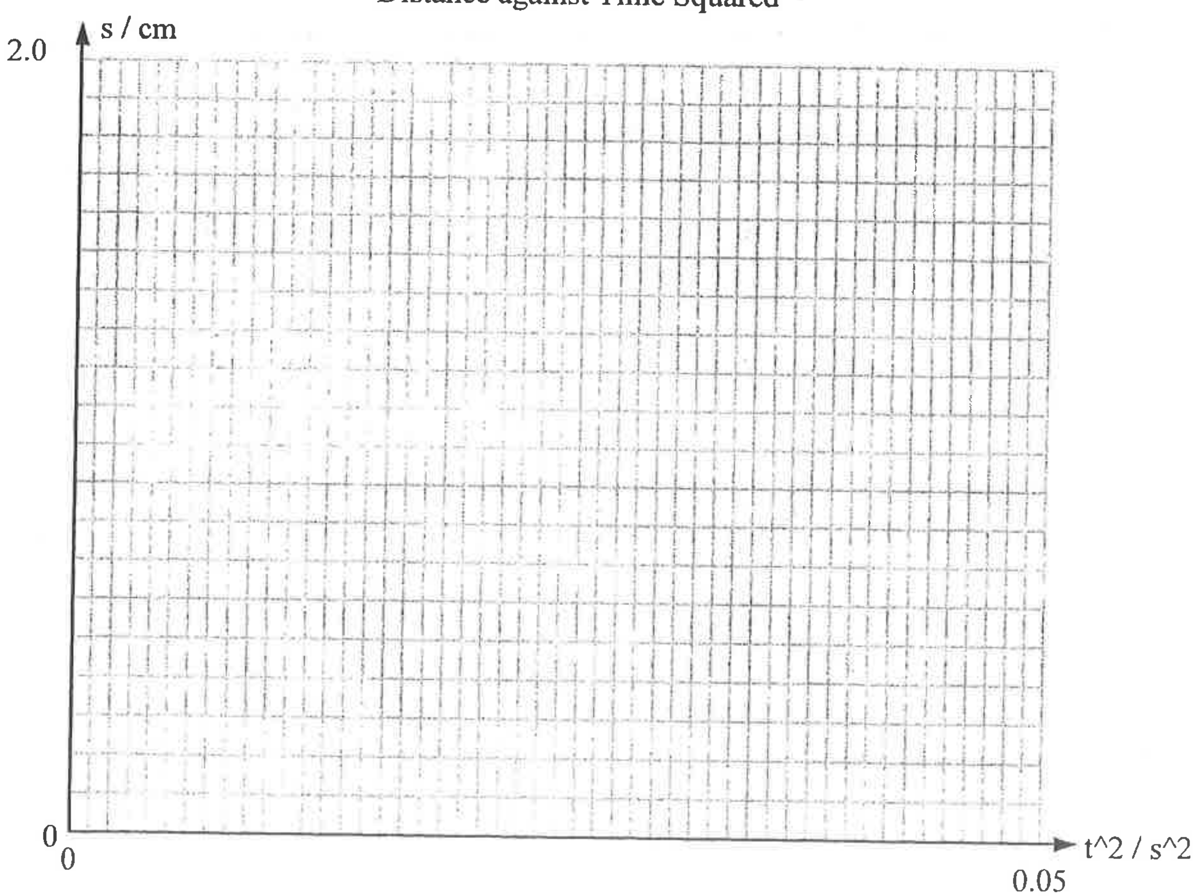
If we measure the time for five successive oscillations and get $2.4 \pm 0.1\text{s}$, what is the final answer (with an absolute uncertainty) for the period? What if we measure 20 oscillations and get a time of $9.4 \pm 0.1\text{s}$?

10. A computer interface is used to measure the position $(s/\mathrm{cm})$ of an object under uniform acceleration $(a/\mathrm{cm\,s^{-2}})$ as a function of time $(t)$. The uncertainty in the time measurement is very small, about $\Delta t = \pm 0.0001\mathrm{s}$, and so you can ignore it, while the uncertainty in the distance is significant, where $\Delta s = \pm 0.1\mathrm{cm}$. Here are the data.

| Time $t/\mathrm{s}$ | Time$^2$ $t^2/\mathrm{s}^2$ | Distance $s/\mathrm{cm}$ |
|---|---|---|
| 0.0000 | | 0.0 |
| 0.0493 | | 0.1 |
| 0.1065 | | 0.5 |
| 0.1608 | | 1.1 |
| 0.1989 | | 1.7 |

The motion of the body can be described by the equation $s = (1/2)at^2 = mt^2$ where the constant $m = 0.5a$. A graph of distance against the square of time should have a slope of "$m$". Acceleration is determined by first finding the slope or gradient, where $a = 2 \times slope$. Use the above data and graph *distance* against *time squared*, construct uncertainty bars for the distance points, and then determine the best straight line and solve for the acceleration. ~~Do not include the origin in your data points.~~

Distance against Time Squared



11. In an optical experiment a deflected ray of light is measured to be at an angle $\theta = (23 \pm 1)°$. Find the *sine* of this angle and then determine the *minimum* and *maximum* acceptable values of this experimental value. Next, express the experimental result in the form $\tan\theta \pm \Delta(\tan\theta)$.

12. On your graph above (in problem 10) draw the minimum and maximum slopes on graph and determine the range of acceleration based on these two extreme slope values.

13. Density is the ratio of mass to volume, where $\rho = m/V$. What is the *density* of a material if $m = 12.4\,\text{kg}$ and $V = 6.68\,\text{m}^3$? Determine the least uncertainty in the mass and in the volume, and then calculate the uncertainty in the density value. Express the uncertainty in the density $\rho$ as an absolute value $\pm\Delta\rho$, as a fractional ratio $\pm(\Delta\rho/\rho)$, and as a percentage $\pm\Delta\rho\%$.

14. What is the uncertainty in the calculated area of a circle whose radius is determined to be $r = (14.6 \pm 0.5)\,\text{cm}$?

15. An electrical resistor has a 2% tolerance and is marked $R = 1800\,\Omega$. What is the range of acceptable values that the resistor might have? An electrical current of $I = (2.1 \pm 0.1)\,\text{mA}$ flows through the resistor. What is the *uncertainty* in the calculated *voltage* across the resistor where the voltage is given as $V = I\,R$?

16. An accelerating object has an initial speed of $u = (12.4 \pm 0.1)\,\text{m}\,\text{s}^{-1}$ and a final speed of $v = (28.8 \pm 0.2)\,\text{m}\,\text{s}^{-1}$. The time interval for this change in speed is $\Delta t = (4.2 \pm 0.1)\,\text{s}$. Acceleration is defined as $a = (v - u)/\Delta t$. Calculate the *acceleration* and its *absolute uncertainty*.

17. What are the *volume* and its *uncertainty* for a sphere with a radius of $r = (21 \pm 1)\,\text{mm}$?

18. Frequency and period are related as reciprocals. What are the *period* and its *absolute uncertainty* when the frequency of $1.00 \times 10^3\,\text{Hz}$ is known to 2%?

19. Einstein's famous equation relates energy and mass with the square of the speed of light, where $E = mc^2$. What is the *percentage of uncertainty* and the *absolute uncertainty* of the *energy* for a mass $m = 1.00\,\text{kg}$ where the speed of light is $c = 3.00 \times 10^8\,\text{m}\,\text{s}^{-1}$?